Open Quantum Systems Ex 2.

1a) $t_{collapse} = \frac{\pi}{g}$, $t_{revival} = \frac{2\pi}{g}|\alpha|$ → corrected from notes

$\Delta\phi \approx 2g\sqrt{\bar{n}}\,t$

$t_r =$

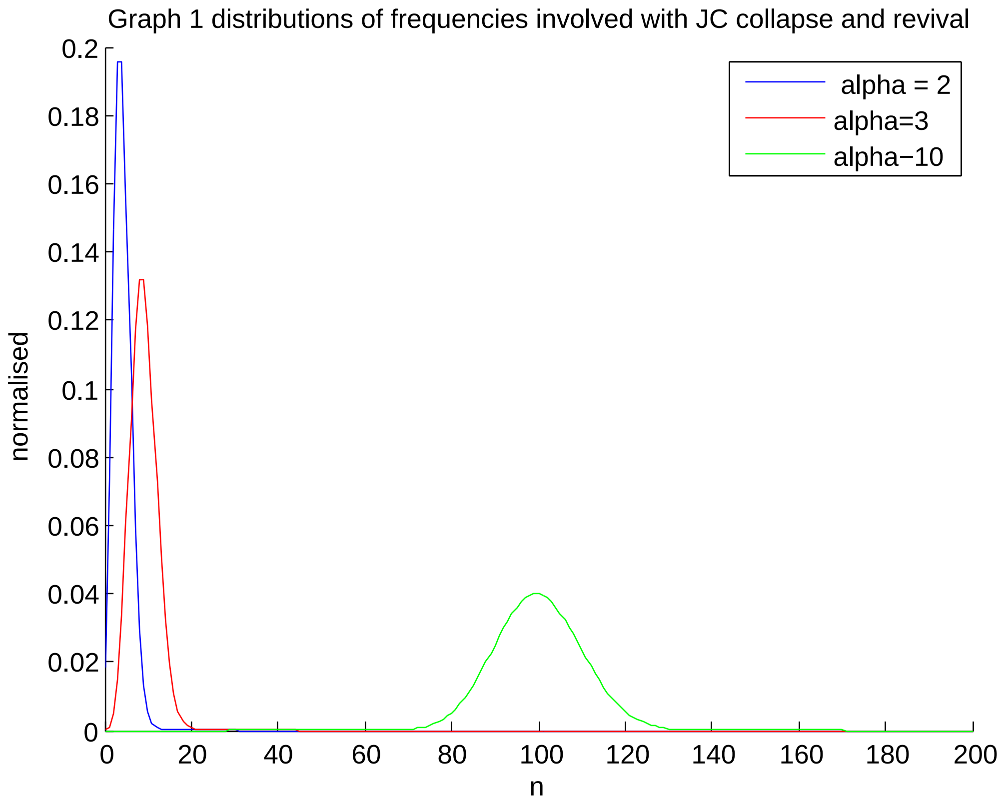
α	2	3	10
t_c	π/g	π/g	π/g
t_r	$4\pi/g$	$6\pi/g$	$20\pi/g$

at increasing α more and more frequencies partake in the collapse & revival. - Graph 1.

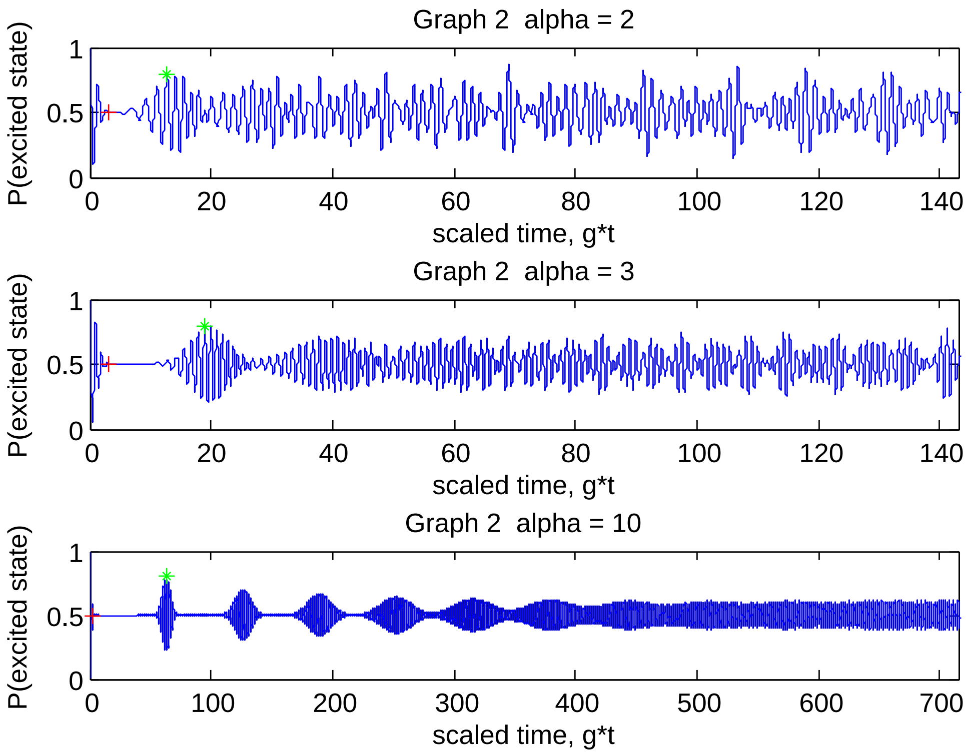
Graph 2 shows the probability to be in an excited state for different sizes of α.
The first red asterix is the expected collapse time and the second ~~red~~ green asterix the expected revival time.
For larger $|\alpha|$ more and more frequencies partake in the interaction so the approximation of $\Delta\phi \propto \sqrt{n}$ is improved and the calculated t_r is in better agreement. $t_{collapse}$ is independent of α and is always a good approximation

b) At longer times more and more frequencies ~~start~~ become distinguishable as the phase difference becomes more significant. For small α only one revival is noticeable before the beating starts, for $\alpha = 10$, 5 or 6 revivals can be seen before the signal reduces to many beatings without recombining

c) the phase of the light source is not relevant and so a thermal source should behave the same as a coherent one



Graph 1 distributions of frequencies involved with JC collapse and revival
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
normalised
0
20
40
60
80
100
120
140
160
180
200
n
alpha = 2
alpha=3
alpha−10


Graph 2 alpha = 2
P(excited state)
1
0.5
0
0
20
40
60
80
100
120
140
scaled time, g*t
Graph 2 alpha = 3
P(excited state)
1
0.5
0
0
20
40
60
80
100
120
140
scaled time, g*t
Graph 2 alpha = 10
P(excited state)
1
0.5
0
0
100
200
300
400
500
600
700
scaled time, g*t

2 a) $\rho = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}$

$$= \frac{1}{2}\begin{pmatrix} 1+P_z & P_x - iP_y \\ P_x + iP_y & 1-P_z \end{pmatrix}$$

$Tr(\rho) = 1 \Rightarrow \rho_{11} = \frac{1}{2}(1+P_z)$, $\rho_{22} = \frac{1}{2}(1-P_z)$

ρ, Hermitian $\Rightarrow \rho_{12} = \frac{1}{2}(P_x - iP_y)$, $\rho_{21} = \frac{1}{2}(P_x + iP_y)$

$$= \frac{1}{2}\left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + P_x\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + P_y\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + P_z\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right]$$

σ_x, σ_y, σ_z

$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$= \frac{1}{2}\left[I + \underline{P} \cdot \underline{\sigma} \right]$$

where $\underline{P} = (P_x, P_y, P_z)$

$P_x = \rho_{12} + \rho_{21}$

$P_y = i(\rho_{12} - \rho_{21})$

$P_z = \rho_{11} - \rho_{22}$

$$\underline{\sigma} = \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{pmatrix}$$

b) <u>von Neumann entropy</u>

$$\rho = \frac{1}{2}\begin{pmatrix} 1+P_z & P_x - iP_y \\ P_x + iP_y & 1-P_z \end{pmatrix}$$

eigenvalues $\left(\begin{vmatrix} 1+P_z-\lambda & P_x - iP_y \\ P_x+iP_y & 1-P_z-\lambda \end{vmatrix} = 0 \Rightarrow (1+P_z-\lambda)(1-P_z-\lambda) - (P_x - iP_y)(P_x+iP_y) = 0\right)$

$$\begin{vmatrix} \rho_{11}-\lambda & \rho_{12} \\ \rho_{21} & \rho_{22}-\lambda \end{vmatrix} = 0 \Rightarrow (\rho_{11}-\lambda)(\rho_{22}-\lambda) - \rho_{21}\rho_{12} = 0$$

$$\lambda^2 - (\rho_{11}+\rho_{22})\lambda + \rho_{11}\rho_{22} - \rho_{21}\rho_{12} = 0$$

$$\lambda^2 - Tr(\rho)\lambda + det(\rho) = 0$$

$Tr(\rho) = 1$ $\therefore$ $\lambda^2 - \lambda + det(\rho) = 0$

$$\lambda = \frac{1 \pm \sqrt{1 - 4\times\frac{1}{4}(1-|P|^2)}}{2}$$

$$\det(\rho) = \frac{1}{4}(\rho_{11}\rho_{22} - \rho_{21}\rho_{12})$$

$$= \frac{1}{4}(1+P_z)(1-P_z) - \frac{1}{4}(P_x - iP_y)(P_x + iP_y)$$

$$= \frac{1}{4}\left(1 - P_z^2 - P_x^2 - P_y^2\right)$$

$$= \frac{1}{4}(1 - |P|^2)$$

$$\lambda_\pm = \frac{1}{2} \pm \frac{1}{2}\sqrt{|P|^2} = \frac{1}{2} \pm \frac{1}{2}|P|$$

von Neumann entropy $\quad S = -\mathrm{tr}(\rho \ln \rho)$

in eigenbasis $\quad S = -\sum_i \lambda_i \log_2 \lambda_i$

$=$

for TLS $\quad S = -\lambda_+ \log_2 \lambda_+ - \lambda_- \log_2 \lambda_-$

$$= -\frac{1}{2}\left[(1+|P|)\log_2\left(\frac{1+|P|}{2}\right) + (1-|P|)\log_2\left(\frac{1-|P|}{2}\right)\right]$$

the entropy ~~for tl~~ is directly related to the distance from the centre of the Bloch sphere.

~~a the surface the system has eigenvalues of that are degenerate. $\lambda_\pm = \frac{1}{2}$, $S =$~~

~~surface $|P| = 1$~~ $\cancel{S = -\frac{1}{2}[\frac{2}{2}\log_2\frac{2}{2} + 0\log(0)]} =$

surface of Bloch sphere $|P| = 1$

$$S = -\frac{1}{2}\left[2\times\log_2 1 + 0\times\log_2 0\right] = 0$$

Centre of Bloch sphere $|P| = 0$

$$S = -\frac{1}{2}\left[1\times\log_2\left(\frac{1}{2}\right) + 1\times\log_2\frac{1}{2}\right]$$

$$= -\frac{1}{2}\times -2 = 1$$

c) Schmidt decomposition

$$|\psi\rangle = \sum_{jk} a_{jk} |j\rangle|k\rangle$$

using singular value decomposition. $a = udv$

where d is a diagonal matrix and u & v are unitary matrices

$$|\psi\rangle = \sum_{ijk} u_{ji} d_{ii} v_{ik} |j\rangle|k\rangle$$

define $|i_A\rangle \equiv \sum_j u_{ji} |j\rangle$, $|i_B\rangle = \sum_k v_{ik} |k\rangle$, $\lambda_i = d_{ii}$

gives $|\psi\rangle = \sum_i \lambda_i |i_A\rangle|i_B\rangle$

d) $\dfrac{|00\rangle + |11\rangle}{\sqrt{2}}$ by observation ~~A~~ $\lambda_1 = \frac{1}{\sqrt{2}}$ $\cancel{|i_A\rangle = |0_A\rangle}$

$$= \sum_k \frac{1}{\sqrt{2}} |k\rangle|k\rangle$$

$$\frac{|00\rangle + |01\rangle + |10\rangle + |11\rangle}{2} = (a|0_A\rangle + b|1_A\rangle)(c|0_B\rangle + d|1_B\rangle)$$

$$= ac|00\rangle + ad|01\rangle + bc|10\rangle + bd|11\rangle$$

~~arc~~

this has ~~solutions~~ a solution. $a = b = c = d = \frac{1}{\sqrt{2}}$

$$\therefore \frac{|00\rangle + |01\rangle + |10\rangle + |11\rangle}{2} = \frac{1}{\sqrt{2}}(|0_A\rangle + |1_A\rangle) \frac{1}{\sqrt{2}}(|0_B\rangle + |1_B\rangle)$$

$$= |A\rangle|B\rangle \qquad |A\rangle = \frac{(|0\rangle_A + |1\rangle_A)}{\sqrt{2}}$$

$$|B\rangle = \frac{(|0\rangle_B + |1\rangle_B)}{\sqrt{2}}$$