Open quantum systems (Nir Bar-Gill, 77541)

Problem set 4, due May 28

1. Master equation for a two-level system (TLS) with decay (see, e.g., "Exploring the Quantum"): Consider a TLS, with states $|g\rangle$ and $|e\rangle$, and an energy difference ω_{ge}. We assume that the excited state $|e\rangle$ is coupled to the ground state $|g\rangle$ through a continuum of electromagnetic modes, and can therefore decay.

    a. How many jump operators are required to describe the above dynamics? Write them using Pauli operators, and assume the associated rate is γ.
    b. Using the jump operators of (a), write the Lindblad master equation for the system.
    c. From the master equation of (b), write explicitly the differential equations for the time evolution of the populations (ρ_{ee}, ρ_{gg}) and the coherences ($\rho_{eg} = \rho_{ge}^*$). What is the resulting decay rate for the populations? And for the coherences?
    d. Add resonant driving of the TLS with Rabi frequency Ω_0 to the Hamiltonian part of the master equation (you can use the interaction picture/rotating frame and rotating wave approximation for simplicity).
    Solve numerically the master equation, assuming $\gamma = \Omega_0$ and the initial state being $|g\rangle$, and plot the excited state population as function of time (with time in units of $1/\Omega_0$).

2. Quantum jumps and the Monte Carlo wavefunction method (K. Molmer et. al., Monte-Carlo wavefunction method in quantum optics, J. Opt. Soc. Am. B 10, 524-538, 1993):
We will compare question 1, describing the dynamics of a driven TLS in the presence of decay, to the dynamics obtained through solving the wavefunction dynamics with random quantum jumps.

    a. Solve numerically the dynamics of a driven TLS (without decay, in the rotating wave approximation), with Rabi frequency Ω_0 and no detuning ($\delta = 0$), starting from the ground state. Plot the excited state population as a function of time (in units of $1/\Omega_0$) and compare to the analytical solution.
    b. Add to the numerical solution in (a), at each time step dt of the simulation, a probability $\gamma * dt * P_e$ (P_e being the excited state population) to undergo a quantum jump (decay, spontaneous emission) from the excited state to the ground state. After each such jump restart the dynamics as in (a), from the ground state (if no jump occurred, simply continue the regular dynamics). Plot the results assuming $\gamma = \Omega_0$.
    c. Ensemble average over 100, 1000, 10000 runs of (b) and plot. Compare to the results of question 1. Do they agree?
    d. Now add a dissipative term to the Hamiltonian with the form $-\frac{i\hbar\gamma}{2}|e\rangle\langle e|$. Solve numerically (without quantum jumps), normalizing the wavefunction to 1 after each

time step. Plot the results and compare to (a). Explain the differences. Find the analytical expression for the normalization factor after each time step.

e. Add quantum jumps to (d), as was done in (b). Ensemble average over 100, 1000, 10000 runs, and plot the results. Compare to question 1.
f. Discuss the physical interpretation of the method. What is the timescale of the quantum jump itself? Why must we have non-Hermitian evolution when no jump occurred? Give an interpretation based on the fact that not measuring decay during a time-step is in fact a partial measurement. How does this relate to the normalization factor from (d)?
g. Find and describe an experiment in which quantum jumps have been measured (look for papers, any physical system is acceptable). What was the observed timescale between jumps?