$$(x_1 + x_2 + x_3 + \dots x_m)^n = \sum_{k_1+k_2+k_3=n} \left(\frac{n}{k_1, k_2, k_3 \dots k_m}\right) x_t^{k_t}$$

$$\binom{n}{k_1, k_2, \dots k_m} = \frac{n!}{k_1! k_2! k_3! \dots k_m!}$$

$$\exp(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\hat{x}_\phi = \frac{\hat{N}_{12}}{|\alpha_L|\sqrt{2}} = \left[\hat{a}(\exp - i\phi) + a^\dagger \exp(i\phi)\right] / \sqrt{2}$$

define

$$\frac{\hat{a}_\phi}{\sqrt{2}} = d$$

$$\frac{\hat{a}^\dagger_\phi}{\sqrt{2}} = c$$

$$\exp\left(-(x_\phi - \hat{x}_\phi)^2 / 2\sigma^2\right) = \exp\left[-\left(x_\phi - \frac{\hat{a}_\phi}{\sqrt{2}} - \frac{\hat{a}^\dagger_\phi}{\sqrt{2}}\right)^2 / 2\sigma^2\right]$$

$$(x_\phi - \hat{c} - \hat{d})^2 = x_\phi^2 + \hat{c}^2 + \hat{d}^2 - 2x_\phi\hat{c} - 2x_\phi\hat{d} + \hat{c}\hat{d} + \hat{d}\hat{c}$$

$$\exp\left(-(x_\phi - \hat{x}_\phi)^2 / 2\sigma^2\right) = \sum_{n=0}^{\infty} \frac{1}{n!}\left[-\frac{(x_\phi - \hat{x}_\phi)^2}{2\sigma^2}\right]^n = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\, 2\sigma^{2n}} \left(x_\phi^2 + \hat{c}^2 + \hat{d}^2 - 2x_\phi\hat{c} - 2x_\phi\hat{d} + \hat{c}\hat{d} + \hat{d}\hat{c}\right)^n$$

$$= \sum_{0}^{\infty} \frac{(-1)^n}{n!\, 2\sigma^{2n}} \left(\sum \frac{n!\, x_\phi^{2k_1}}{k_1! k_2! k_3! k_4! k_5! k_6}\right)$$

NORMAL ORDERING put $\hat{c}$ to LHS of $\hat{d}$