Notebook No.__________ מחברת מס.

Project: Nonlinear eqs. for a system with kinetic inductance פרוייקט

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$$\mathcal{L} = \frac{(L_0 + L_2 \dot{Q}^2)\dot{Q}^2}{2} - \frac{Q^2}{2C}$$

לגרנז'יאן:

$$\frac{d}{dt}\frac{\partial}{\partial \dot{Q}}\left(\frac{L_2 \dot{Q}^4}{2}\right) = \frac{d}{dt}\left(2L_2\dot{Q}^3\right) = 6L_2\dot{Q}^2\ddot{Q}$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{Q}}\right) = \frac{\partial \mathcal{L}}{\partial Q} \Rightarrow L_0\ddot{Q}\left(1 + \underbrace{6\frac{L_2}{L_0}}_{\alpha}\dot{Q}^2\right) = -\frac{Q}{C}$$

$$\ddot{Q} + \frac{Q}{L_0 C}(1+\alpha\dot{Q}^2)^{-1} \Rightarrow \ddot{Q} \approx -\frac{Q}{L_0C} + \frac{\alpha}{L_0C}\dot{Q}^2 Q$$

אם $\alpha \ll 1$ (נכון?)

$$F_d(t) = \ddot{q}_i + \omega_0^2 q - \alpha\omega_0^2 \underset{q\dot{q}^2}{\wedge} + \frac{\omega}{2Q}\dot{q} \pm K_c(q_1 - q_2)$$

[illegible]

$$\omega_0^2 \equiv \frac{1}{L_0C}$$

SVE approximation: נניח

$$q_i(t) = \tilde{q}_i(t)e^{i\omega t}, \quad F_d(t) = f_d e^{i\omega t}$$

אשר $\tilde{q}(t)$ משתנה לאט: $|\dot{\tilde{q}}| \ll \omega$

$$\dot{q}_i = \dot{\tilde{q}}_i(t)e^{i\omega t} + i\omega\tilde{q}_i(t)e^{i\omega t}$$

$$\ddot{q}_i = \ddot{\tilde{q}}_i(t)e^{i\omega t} + 2i\omega\dot{\tilde{q}}_i(t)e^{i\omega t} - \omega^2 q_i(t)e^{i\omega t}$$

[illegible]

$$\dot{q}^2 = \frac{1}{4}\left(\dot{\tilde{q}}_i e^{i\omega t} + i\omega\tilde{q}_i e^{i\omega t} + \dot{\tilde{q}}_i e^{-i\omega t} - i\omega\tilde{q}_i e^{-i\omega t}\right)\left(\dot{\tilde{q}}_i e^{i\omega t} + i\omega\tilde{q}_i e^{i\omega t} + \dot{\tilde{q}}_i e^{-i\omega t} - i\omega\tilde{q}_i e^{-i\omega t}\right)$$

$$= \frac{1}{4}\Big(\dot{\tilde{q}}^2 e^{2i\omega t} + i\omega\tilde{q}\dot{\tilde{q}}e^{2i\omega t} + \dot{\tilde{q}}^2 - i\omega\dot{\tilde{q}}\tilde{q} + i\omega\tilde{q}\dot{\tilde{q}}e^{2i\omega t} - \omega^2\tilde{q}^2 e^{2i\omega t} + i\omega\tilde{q}\dot{\tilde{q}} + \omega^2\tilde{q}^2$$

$$+ \dot{\tilde{q}}\tilde{q} + i\omega\dot{\tilde{q}}\tilde{q} + \dot{\tilde{q}}^2 e^{-2i\omega t} - i\omega\tilde{q}\dot{\tilde{q}}e^{-2i\omega t} - i\omega\tilde{q}\dot{\tilde{q}} + \omega^2\tilde{q}^2 - i\omega\tilde{q}\dot{\tilde{q}}e^{-2i\omega t} - \omega^2\tilde{q}^2 e^{-2i\omega t}\Big)$$

$$= \frac{1}{4}\left(\dot{\tilde{q}}^2 e^{2i\omega t} + c.c + 2i\omega\underset{\tilde{q}\dot{\tilde{q}}}{\wedge}e^{2i\omega t} + c.c + 2\dot{\tilde{q}}^2 - \omega^2\tilde{q}^2 e^{2i\omega t} + c.c + 2\omega^2\tilde{q}^2\right)$$

$$\dot{q}^2 q = \frac{\tilde{q}}{8}(\quad)(e^{i\omega t} + e^{-i\omega t})$$

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roject: Steady-State Solution for osc. with $\dot{q}^2 q$ non-linearity פרוייקט

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$$= \frac{\tilde{q}}{8}\Big(\tilde{\dot{q}}^2 e^{3i\omega t} + 2i\omega e^{3i\omega t} + 2\tilde{\dot{q}}^2 e^{i\omega t} - \omega^2 \tilde{q}^2 e^{3i\omega t} + 2\omega^2 \tilde{q}^2 e^{i\omega t}$$
$$+ \tilde{\dot{q}}^2 e^{i\omega t} + 2i\omega e^{i\omega t} + 2\tilde{\dot{q}} e^{-i\omega t} - \omega^2 \tilde{q} e^{i\omega t} + 2\omega^2 \tilde{q}^2 e^{-i\omega t} + c.c\Big)$$

$$= \frac{\tilde{q}}{4}\Big[\tilde{\dot{q}}^2 \cos(3\omega t) - 2\omega \sin(3\omega t) + \tilde{\dot{q}}^2 \cos(\omega t) - \omega^2 \tilde{q}^2 \cos(3\omega t) + 4\omega^2 \tilde{q}^2 \cos(\omega t)$$
$$+ \tilde{\dot{q}}^2 \cos(\omega t) - 2\omega \sin(\omega t) - \omega^2 \tilde{q}^2 \cos(\omega t)\Big]$$

$(A + A^*)(B + B^*)$

$= AB + AB^* + A^*B + A^*B^*$

$= (AB + AB^* + c.c)$

$(A + A^* + C)(B + B^*)$

$= (A + A^*)(B + B^*)$

$+ C(B + B^*)$

$= (AB + AB^* + c.c$

$+ CB + CB^*)$

$$= \frac{\tilde{q}}{8}\Big(\tilde{\dot{q}}^2 e^{3i\omega t} + 2i\omega \tilde{q}\tilde{\dot{q}} e^{3i\omega t} + 2\tilde{\dot{q}}^2 e^{i\omega t} - \omega^2 \tilde{q}^2 e^{3i\omega t} + 2\omega^2 \tilde{q}^2 e^{i\omega t}$$
$$+ \tilde{\dot{q}}^2 e^{i\omega t} + 2i\omega \tilde{q}\tilde{\dot{q}} e^{i\omega t} + 2\tilde{\dot{q}}^2 e^{-i\omega t} - \omega^2 \tilde{q}^2 e^{i\omega t} + 2\omega^2 \tilde{q}^2 e^{i\omega t} + c.c\Big)$$

$$= \frac{\tilde{q}}{4}\Big[\tilde{\dot{q}}^2 \cos(3\omega t) - 2\omega \tilde{q}\tilde{\dot{q}} \sin(3\omega t) + \ldots +$$

$$= \frac{\tilde{q}}{8}\Big(\tilde{\dot{q}}^2 e^{3i\omega t} + c.c + 2i\omega \tilde{q}\tilde{\dot{q}} e^{3i\omega t} + c.c - \omega^2 \tilde{q}^2 e^{3i\omega t} + c.c + \tilde{\dot{q}}^2 e^{i\omega t} + c.c + 2i\omega \tilde{q}\tilde{\dot{q}} e^{i\omega t} + c.c$$
$$- \omega^2 \tilde{q}^2 e^{i\omega t} + c.c$$
$$+ 2\tilde{\dot{q}}^2 e^{i\omega t} + 2\tilde{\dot{q}}^2 e^{-i\omega t} + 2\omega^2 \tilde{q}^2 e^{i\omega t} + 2\omega^2 \tilde{q}^2 e^{-i\omega t}\Big)$$

$$= \frac{\tilde{q}}{4}\Big[\tilde{\dot{q}}^2 \cos(3\omega t) - 2\omega \tilde{q}\tilde{\dot{q}} \sin(3\omega t) - \omega^2 \tilde{q}^2 \cos(3\omega t) + 3\tilde{\dot{q}}^2 \cos(\omega t) - 2\omega \tilde{q}\tilde{\dot{q}} \sin(\omega t) + \omega^2 \tilde{q}^2 \cos(\omega t)\Big]$$

⇐ לפי דימוי והזנחת תלויות [illegible]:

$$f_d = \tilde{\ddot{q}} + \tilde{q}\left[\omega_0^2 - \omega^2 + i\omega\Gamma\right] + \tilde{\dot{q}}\left[2i\omega + \Gamma\right] + \tilde{q}\tilde{\dot{q}}^2\left[-\alpha\omega_0^2 \cdot \frac{3}{8}\right] + \tilde{q}^2\tilde{\dot{q}}\left[\frac{i\omega}{4} \cdot -\alpha\omega_0^2\right] + \tilde{q}^3\left[\frac{-\alpha\omega_0^2\omega^2}{8}\right]$$

⇒ [illegible]

$$f_d = \tilde{q}\left[\omega_0^2 - \omega^2 + i\omega\Gamma\right] - \tilde{q}^3\left[\frac{\alpha\omega_0^2\omega^2}{8}\right]$$

$$\frac{\alpha\omega_0^2\omega^2}{8}\tilde{q}^3 - \tilde{q}\left[\omega_0^2 - \omega^2 + i\omega\Gamma\right] + f_d = 0$$

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