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Supporting Online Material for

High-NOON States by Mixing Quantum and Classical Light

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Published 14 May 2010, *Science* **328**, 879 (2010)
DOI: 10.1126/science.1188172

This PDF file includes:

SOM Text

Fig. S1

References

These supporting online materials contain a complete theoretical analysis of our experiment. In Sec. S1 we develop the analytical model used for calculating the theoretical curves in Fig. 3 of the main text. A brief discussion of the overall setup transmission and its importance is given in Sec. S2. In Sec. S3 we illustrate the feasibility of nine-photon NOON state generation in our setup using the theoretical model.

S1 Analytical Model

The experimental results in Fig. 3 of the main text contain multiphoton coincidence rates as a function of the Mach-Zehnder (MZ) phase φ. The probability for detecting N_1 photons in D_1 and N_2 photons in D_2 simultaneously (*S1*) (see experimental setup in Fig. 2 of the main text) is given by

$$p_{N_1,N_2}(\varphi) = \mathrm{Tr}\left[\hat{U}(\varphi)\,|\alpha\rangle\langle\alpha|_a \otimes |\xi\rangle\langle\xi|_b \hat{U}^{\dagger}(\varphi)\,\hat{\pi}^1_{N_1} \otimes \hat{\pi}^2_{N_2}\right], \tag{S1}$$

where $\hat{U}(\varphi)$ is a unitary operator describing the MZ using angular momentum notation (*S2*)

$$\hat{U}(\varphi) = e^{\imath(\pi/2)\hat{J}_x} e^{-\imath\varphi\hat{J}_z} e^{-\imath(\pi/2)\hat{J}_x}. \tag{S2}$$

The coherent and down-conversion input states $|\alpha\rangle$,$|\xi\rangle$ are defined in the conventional way,

$$|\alpha\rangle = \sum_{n=0}^{\infty} e^{-\frac{1}{2}|\alpha|^2} \frac{\alpha^n}{\sqrt{n!}} |n\rangle, \quad \alpha = |\alpha| e^{\imath\phi_{cs}}$$

$$|\xi\rangle = \frac{1}{\sqrt{\cosh r}} \sum_{m=0}^{\infty} (-1)^m \frac{\sqrt{(2m)!}}{2^m m!} (\tanh r)^m |2m\rangle, \tag{S3}$$

and $\hat{\pi}_N^{1(2)}$ are the positive-operator-valued measures (POVM) (*S3*) of detectors $D_{1(2)}$ defined,

$$\hat{\pi}_N^{1(2)} = \sum_{k=0}^{\infty} \theta_{N,k} |k\rangle\langle k|_{1(2)}, \qquad \theta_{N,k} = [C_M \cdot L(\eta)]_{N,k}\,. \tag{S4}$$

The POVM introduces experimental imperfections via two matrices, C_M and $L(\eta)$. Using the abstract loss model (*S4*) we account for all the sources of loss in the experiment using a single parameter η representing the overall transmission. The non-zero loss matrix elements are given by

$$[L(\eta)]_{n,m} = \binom{m-1}{n-1} \eta^{n-1} (1-\eta)^{m-n} \qquad n, m = 1, 2 \ldots\,. \tag{S5}$$

Each of our photon number resolving detectors consists of two (D_1) or three (D_2) avalanche photodiodes (APD) such that the incoming photons are distributed equally between them. This construction is reflected in the POVM via the matrix C_M, where M represents the number of sub-detectors. The non-zero matrix elements of C_M for $M = 2, 3$ are given by

$$\begin{aligned} &[C_2]_{1,1} = 1; \\ &\begin{cases} [C_2]_{2,k} = \dfrac{1}{2^{k-2}}; \\ \\ [C_2]_{3,k} = 1 - [C_2]_{2,k}; \end{cases} \qquad k = 2, 3 \ldots \end{aligned} \tag{S6}$$

$$[C_3]_{1,1} = 1;$$

$$[C_3]_{2,2} = 1;$$

$$\begin{cases} [C_3]_{2,k} = \frac{1}{3^{k-2}}; \\ [C_3]_{3,k} = 1 - \frac{1}{3^{k-2}} \times (2^{k-1}); \qquad k = 3, 4 \ldots \\ [C_3]_{4,k} = 1 - [C_3]_{2,k} - [C_3]_{3,k}; \end{cases} \tag{S7}$$

Finally, to obtain the measured count rate, the probability of Eq. (*S1*) should be multiplied by the repetition rate of 80.07 MHz.

S2 Overall Transmission

The overall transmission η is an important experimental factor which determines the obtainable visibility of super-resolution. In our setup, we find $\eta = 0.12$ by inserting down-conversion only and measuring the coincidence to singles ratio. Roughly, this transmission has three multiplicative contributions $\eta \sim \eta_1 \times \eta_2 \times \eta_3$. The first contribution $\eta_1 \sim 0.5$ results from the fiber pair-coupling ratio. The band-pass filter induces an additional effective loss, $\eta_2 \sim 0.5$, and finally the detector efficiencies together with various coating imperfections contribute $\eta_3 \sim 0.5$.

Most of the photons in our experiment originate from the coherent (classical) light source which is practically unlimited in intensity. In fact, it can be shown that $\gamma_N \sim N/2$ for large N where γ_N is the optimal pair amplitude ratio for a given photon number N. Thus, the coherent state's two photon probability is approximately $N^2/4$ times higher than that of the SPDC i.e. the higher the value of N the larger the ratio of classical to quantum resources. Therefore, measuring larger states does not require a brighter SPDC source. Thus, the only limiting factor is the overall transmission, η. Simulations show that using an overall transmission of $\eta = 0.5$, nine photon entanglement is readily observable even with the current, relatively modest, SPDC flux. In this respect, our experiment highlights the need for high purity SPDC sources which can be spectrally mode matched to a coherent state. Improved transmission can be obtained by way of SPDC generation in separable spectral modes (*S5*-*S7*) allowing removal of the 3nm band-pass filter. In addition, improved single mode coupling of the photon pairs and use of high efficiency photon number resolving detectors (*S8*-*S9*) are instrumental.

S3 Nine Photon NOON State

In this section we show that our setup is capable of generating nine photon NOON states when using a reasonable overall transmission $\eta = 0.5$. This transmission could be obtained by using improved photon number resolving detectors, improved pair coupling to the fiber and using down-conversion which is separable in frequency modes. The analytical model of Sec. S1 allows us to investigate the behavior of our setup using any chosen overall transmission η. We note that for $N = 9$, the pair amplitude ratio is $\gamma_9 = 3.9$ implying that the coherent light photon-pair probability is $\gamma_9^2 = 15.3$ times higher than for the down-conversion. In supplementary Fig. S1 we show the nine-photon coincidence rate employing our current down-conversion flux while using $\eta = 1$ (Fig. S1A) or $\eta = 0.5$ (Fig. S1B). This illustrates that our setup allows measurement of nine photon super-resolution originating from NOON states when using a feasible transmission.

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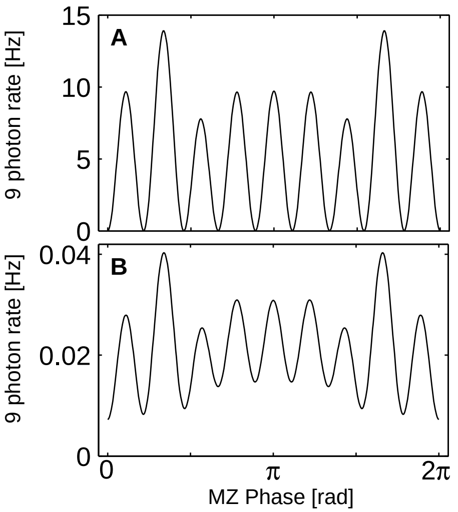
Figure S1 Simulation of nine-photon coincidence rates showing super-resolution. Nine-fold coincidence rates vs. MZ phase using two overall transmission values. In this case we chose $N_1 = 6, N_2 = 3$. **A**, with $\eta = 1$, **B**, with $\eta = 0.5$.



A
B
15
10
5
0
0.04
0.02
0
9 photon rate [Hz]
9 photon rate [Hz]
0
π
2π
MZ Phase [rad]